**MATHEMATICS SPECIALIST**

**MAWA Year 12 Examination 2017**

**Calculator-assumed**

# Marking Key

© MAWA, 2017

**Licence Agreement**

This examination is Copyright but may be freely used within the school that purchases this licence.

* The items that are contained in this examination are to be used solely in the school for which they are purchased.
* They are not to be shared in any manner with a school which has not purchased their own licence.
* The items and the solutions/marking keys are to be kept confidentially and not copied or made available to anyone who is not a teacher at the school. Teachers may give feedback to students in the form of showing them how the work is marked but students are not to retain a copy of the paper or marking guide until the agreed release date stipulated in the purchasing agreement/licence.

The release date for this exam and marking scheme is

* **the end of week 1 of term 4, 2017**

**Question 8 (a)**

|  |
| --- |
| **Solution** |
| Also  In addition,  and |
| **Specific behaviours** |
| 🗸 🗸 calculates the modulus and argument of  correctly  🗸 relates the modulus and argument of  to those of |

**Question 8 (b)**

|  |
| --- |
| **Solution** |
| Now    As  this means that |
| **Specific behaviours** |
| 🗸 uses De Moivre’s theorem to express answer as an exponential  🗸 simplifies answer to form of an exponential of small argument  🗸 obtains correct answer in Cartesian form |

**Question 9 (a)**

|  |
| --- |
| **Solution** |
| Let  gm denote the average weight of a randomly chosen coffee bean  Then  and so  and  so that  Solving equations (A) and (B) gives  and |
| **Specific behaviours** |
| 🗸 obtains equation (A)  🗸 obtains equation (B)  🗸 solves for  and  correctly |

**Question 9 (b)**

|  |
| --- |
| **Solution** |
| If the 10 beans in the random sample weigh more than 1.2 gm, then  where  is the average weight of beans in the sample.  Now  i.e. |
| **Specific behaviours** |
| 🗸 🗸 uses mean and correct standard deviation for  🗸 obtains correct answer |

**Question 9 (c)**

|  |
| --- |
| **Solution** |
| Now  But  and  Hence the required probability is 0.67. |
| **Specific behaviours** |
| 🗸 obtains correct bounds for the probability in terms of  🗸 evaluates limits correctly  🗸 derives correct answer |

**Question 10 (a)**

|  |
| --- |
| **Solution** |
| If    Then we have  Hence we deduce that |
| **Specific behaviours** |
| 🗸 Combines the two given fractions correctly  🗸 Expands the parts of the numerator  🗸 Writes the three correct equations for the constants  🗸 Solves for the constants |

**Question 10 (b)**

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸🗸 integrates the two partial fractions correctly  🗸 simplifies the solution to the required form |

**Question 11(a)**

|  |
| --- |
| **Solution** |
| Substituting  into the two vector equations  and |
| **Specific behaviours** |
| 🗸 correct answer for |

**Question 11(b)**

|  |
| --- |
| **Solution** |
| Differentiating with respect to  gives |
| **Specific behaviours** |
| 🗸 differentiates each position vector correctly |

**Question 11(c)**

|  |
| --- |
| **Solution** |
| Since the velocity vector of each plane is independent of , the velocity of each plane is constant, indicating motion in a straight line.  The speed of plane A is  while the speed of plane B is 3 km/min.  The flight path of plane A is trending upwards while the flight path of plane B is trending downwards. |
| **Specific behaviours** |
| 🗸 remarks that the velocity of each plane is constant/linear flight path  🗸 calculates the speeds of the two planes  🗸 states the vertical direction of each plane. |

**Question 11(d)**

|  |
| --- |
| **Solution** |
| Using a CAS calculator, determine an expression for the distance  Graph the expression using the graphing app    Use the ‘Analysis’ function on the CAS  To determine the minimum distance between the two planes.  Min distance = 14.825 km  Occurs when t=3.71 approx |
| **Specific behaviours** |
| 🗸 🗸 draws a graph with the appropriate shape  🗸 calculates the correct min distance and gives the corresponding time |

**Question 12 (a)**

|  |
| --- |
| **Solution** |
| The function is a quadratic with a maximum at  This corresponds to half the population. |
| **Specific behaviours** |
| 🗸 recognizes the quadratic nature of the derivative function |

**Question 12 (b)**

|  |
| --- |
| **Solution** |
| Separating the variables in the differential equation gives  and so  Therefore  for some constant c. |
| **Specific behaviours** |
| 🗸 separates the variables correctly  🗸 integrates each side correctly |

**Question 12 (c)**

|  |
| --- |
| **Solution** |
| Substituting into the expression in (b) gives  and so  Hence |
| **Specific behaviours** |
| 🗸 obtains correct value for  🗸 derives the correct equation |

**Question 12 (d)**

|  |
| --- |
| **Solution** |
| Since  then  and  Then |
| **Specific behaviours** |
| 🗸 takes exponentials correctly  🗸 derives correct formula for |

**Question 12 (e)**

|  |
| --- |
| **Solution** |
| When  then  So it takes about 75.4 days for 95% of the population to be infected. |
| **Specific behaviours** |
| 🗸 derives an equation for the required time  🗸 obtains correct answer |

**Question 13 (a)(i) (11 marks)**

|  |
| --- |
| **Solution** |
| If  Thus |
| **Specific behaviours** |
| 🗸 Rearranges to obtain  in terms of  🗸 Interchanges  and  in formula  🗸 Deduces the form of the inverse function |

**Question 13 (a)(ii)**

|  |
| --- |
| **Solution** |
| Domain is ; Range is |
| **Specific behaviours** |
| 🗸 States domain and range correctly |

**Question 13 (b)**

|  |
| --- |
| **Solution** |
| Now  The composite is only defined where  is defined so that  . |
| **Specific behaviours** |
| 🗸 Determines the correct composite function  🗸 Identifies the correct domain of definition. |

**Question 13 (c)**

|  |
| --- |
| **Solution** |
| Recall that  if  and that  for  Then  Case 1:    Case 2:    Case 3:  .  But this is a contradiction as it has been assumed that  Hence the solutions of the equation are |
| **Specific behaviours** |
| 🗸 Draws correct conclusion from case 1  🗸 Solves equation in case 2  🗸 Solves equation in case 3  🗸 Recognises that the apparent solution in case 3 contradicts the assumed range  🗸 Synthesises results from the three cases to deduce the correct solution of the problem |

**Question 14(a)**

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 uses the dot product and expands correctly  🗸 uses and states why it is zero  🗸 deduces the correct result |

**Question 14(b)**

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 determines in terms of  using part (a)  🗸 determines and  in terms of  🗸 sums to find the result =  🗸show this equals |

**Question 14(c)**

|  |
| --- |
| **Solution** |
| The sum of the squares of the diagonals of a rectangle are equal to twice the sum of the squares of any two adjacent sides of the rectangle. |
| **Specific behaviours** |
| 🗸 states a correct interpretation of the result |

**Question 15 (a)**

|  |
| --- |
| **Solution** |
| We want i.e.  Therefore  and so we need to test at least 48 tyres. |
| **Specific behaviours** |
| 🗸 obtains correct inequality  🗸 solves for  correctly |

**Question 15 (b)**

|  |
| --- |
| **Solution** |
| The confidence interval is  where    Hence the confidence interval is  i.e. |
| **Specific behaviours** |
| 🗸 uses correct form for the confidence interval  🗸 obtains correct value for  the margin of error  🗸 obtains correct limits for the confidence interval |

**Question 15 (c)**

|  |
| --- |
| **Solution** |
| The sample does provide some evidence to dispute the manfacturer’s claim because the sample mean is less than 40000.  However, the confidence interval  contains some numbers greater than 40000, and so the evidence for rejecting the claim is not compelling. |
| **Specific behaviours** |
| 🗸 notes that there is some evidence for disputing the claim  🗸 recognises that the evidence for rejecting is not overwhelming. |

**Question 16 (a)**

|  |
| --- |
| **Solution** |
| The graph suggests zeroes at and .  To check this we calculate    and |
| **Specific behaviours** |
| 🗸 estimates zeroes at  and  from a graph  🗸 checks these are correct by direct substitution |

**Question 16 (b)**

|  |
| --- |
| **Solution** |
| Since roots of  are  and  this suggests that  is a factor of .  By long division we observe that      If  Hence the solutions of  are  and . |
| **Specific behaviours** |
| 🗸 🗸 identifies the two linear factors that follow from (a)  🗸 🗸 conducts the long division correctly  🗸 writes down the solution of the quadratic and hence finds correct complex zeros |

**Question 17 (a)**

|  |
| --- |
| **Solution** |
| If  then    and    Hence    as required |
| **Specific behaviours** |
| 🗸 obtains correct expression for  🗸 obtains correct expression for  🗸 completes proof |

**Question 17 (b)**

|  |
| --- |
| **Solution** |
| Since  so |
| **Specific behaviours** |
| 🗸 obtains correct expression for the velocity  🗸 deduces the correct value for the constant |

**Question 17 (c)**

|  |
| --- |
| **Solution** |
| The spring is stationary when  i.e. when  i.e. when  The least positive solution of this equation is  So the spring is first stationary at time  seconds approximately |
| **Specific behaviours** |
| 🗸 obtains equation for stationary times  🗸 calculates the correct answer |

**Question 17 (d)**

|  |
| --- |
| **Solution** |
| From part (c) the spring is stationary when  The solutions are then  where  is any non-negative integer.  These form an arithmetic sequence with common difference is  . |
| **Specific behaviours** |
| 🗸 obtains general solution for the stationary times  🗸 states the correct common difference between these times |

**Question 17 (e)**

|  |
| --- |
| **Solution** |
| At a local maximum of  we have  This occurs when  where  Now  We know that  will be positive if  is even and negative if  is odd  Hence the local maxima of  correspond to  even.  So the local maxima of  are given by  where  is any non-negative integer.    These form a geometric sequence in which the common ratio is |
| **Specific behaviours** |
| 🗸 evaluates  at the stationary point  🗸 shows that every second stationary point is a local maximum  🗸 realises the maxima correspond to even values of  🗸 observes the maxima values constitute a GP and states the common ratio |

**Question 18 (a)**

|  |
| --- |
| **Solution** |
| For the region  the parabola is above the x-axis so the required area is |
| **Specific behaviours** |
| 🗸 states correct limits of integration  🗸 writes a correct expression for the area  🗸 integrates all the terms correctly  🗸 substitutes in values to determine the area |

**Question 18 (b)**

|  |
| --- |
| **Solution** |
| If rotating about -axis have that    To compute volume around y-axis need to evaluate  The second volume is determined by calculating the volume  generated by rotating the right-hand branch of the parabola about the axis and subtracting the volume  formed using the left-hand branch of the parabola.  The parabola is . Hence  Right hand branch is with +sign and is defined by  . Then    Left hand branch is with – sign and is also defined by . Then    Thus volume generated by rotating about the  axis is  Hence we see that |
| **Specific behaviours** |
| 🗸 writes down correct form of integral for first volume  🗸🗸 integrates correctly and hence evaluates to determine the volume  (candidates are expected to use calculator for the manipulation)  🗸 realises that for rotation is about y-axis the required integral is of form  🗸 gives clear statement of strategy adopted to determine the volume  🗸 determines the appropriate formulae for the two branches of the parabola  🗸🗸 for volume  forms correct integral with limits. Evaluates correctly (calc allowed)  🗸🗸 for volume  forms correct integral with limits. Evaluates correctly  🗸 deduces the correct relationship between  and |